

NASA TECH BRIEF

Lewis Research Center



NASA Tech Briefs announce new technology derived from the U.S. space program. They are issued to encourage commercial application. Tech Briefs are available on a subscription basis from the National Technical Information Service, Springfield, Virginia 22151. Requests for individual copies or questions relating to the Tech Brief program may be directed to the Technology Utilization Office, NASA, Code KT, Washington, D.C. 20546.

Indefinite Integrals of Products of Some Exponential and Trigonometric Functions

Two integrals have been developed which are needed to solve certain problems of mathematical physics related to the Bessel and Euler equations. These integrals are:

$$\int x^p \cos(b \ln x) dx = \frac{x^{p+1}}{(p+1)^2 + b^2}$$

$$\left[b \sin(b \ln x) + (p+1) \cos(b \ln x) \right] + c;$$

$$\int x^p \sin(b \ln x) dx = \frac{x^{p+1}}{(p+1)^2 + b^2}$$

$$\left[(p+1) \sin(b \ln x) - b \cos(b \ln x) \right] + c$$

where x is the variable, b is an arbitrary constant, and p is unrestricted. Tables of integrals contain these integrals for $p=0$ and $p=-1$, but not for an unrestricted value of the exponent.

To obtain the above integrals, consider

$$\int x^p e^{ib \ln x} dx = \int x^{ib+p} dx = \frac{x^{ib+p+1}}{(p+1) + ib} + c$$

where $i = \sqrt{-1}$, e is the base of the natural logarithm, and c is the integration constant.

Rationalizing the denominator and expressing the exponential function in terms of the circular functions results in:

$$\int x^p e^{ib \ln x} dx = \frac{[(p+1) - ib] x^{p+1}}{(p+1)^2 + b^2}$$

$$\left[\cos(b \ln x) + i \sin(b \ln x) \right] + c$$

Splitting into the real and the imaginary parts yields the two integrals.

Notes:

1. Integrals of this type have known application in problems using cylindrical coordinates.
2. Documentation may be obtained from:

Technology Utilization Officer
Lewis Research Center
21000 Brookpark Road
Cleveland, Ohio 44135
Reference: B72-10225

Patent status:

No patent action is contemplated by NASA.

Source: W. Rostafinski
Lewis Research Center
(LEW-11493)